# <u>Fundamental Noise Theory</u>

**Carson's Theorem:**

In s single noise event, it can be shown that the "spectral density function, $S_{\mathbf{T}}(\omega)$" of v(t) for an integration time **T** is (see Amnon Yariv's *Optical Electronics*)

$S_{\mathbf{T}}(\omega) \equiv \frac{|V(\omega)|^2}{\pi \mathbf{T}}$ or $S(f) \equiv \frac{2|V(2\pi f)|^2}{\mathbf{T}}$ ,where the frequency domain, V(ω), is the Fourier transform of v(t) in time domain. **T** is the time period.

If there are $N_T$ independent events in the period of **T**, then

$S(\omega) \equiv \frac{N_T |V(\omega)|^2}{\pi \mathbf{T}} = \frac{\overline{N_T} |V(\omega)|^2}{\pi}$ **or** $S(f) \equiv = 2\overline{N_T}|V(2\pi f)|^2$**,** where $\overline{N_T}$ is the rate of event (ie, $N_T/\mathbf{T}$).

Note that:

$$V(\omega) = \int_{-\infty}^{\infty} v(t) e^{-j\omega t} dt$$

$$v(t) = \int_{-\infty}^{\infty} V(\varpi) e^{j\omega t} d\varpi$$

$$p \equiv \int_{0}^{\infty} S_T(\varpi) d\varpi = \frac{1}{T} \int_{-T/2}^{T/2} v(t)^2 dt = \int_{0}^{\infty} \frac{|V(\omega)|^2}{\pi \mathrm{T}} d\varpi$$

$$\Rightarrow S_{\mathrm{T}}(\omega) \equiv \frac{|V(\omega)|^2}{\pi \mathrm{T}}$$

# Types of Noise

The first person to attack noise (caused by atoms) problem is Albert Einstein, which originated from Brownian motions and it was 1905. Similarly, electrical noise is caused by electrons so it is also ubiquitous. There are many kinds of electrical noises including thermal (Johnson) noise, shot noise, 1/f (pink) noise, burst (pop corn) noise and avalanche noise. The following will discuss only the most important noises.

## Shot Noise

Shot noise is proportional to the current through the device and is caused by the random passage of electrons and holes through a potential barrier (without considering collision). Noise is produced because charges are quantized. Any device with a DC current has short noises and the noise amplitude has a Gaussian distribution. In this case, using Carson's theorem, one can get $|i(\omega)| = q$ and $S_i(f) \equiv = 2\overline{N_T}|I(2\pi f)|^2 = 2\overline{N_T}q^2 = 2qI$ **, or**

$$\boxed{\left\langle i_{n,shot}^2 \right\rangle = 2qBI}$$

where $\left\langle i_{n,shot}^2 \right\rangle$ is the root-mean-squared shot noise current, B is the bandwidth, and I is the current through the device.

*Example: Photodiode*

$I_{dc} = 1$ Milli Ampere

$B = 100$ KHz

$T = 300$ K

$\Rightarrow \sqrt{\left\langle i_n^2 \right\rangle} = 5.6\text{nA}$

$\sqrt{\left\langle v_n^2 \right\rangle} = 5.6\mu V$

## Thermal Noise (Johnson Noise or white noise)

Caused by thermal random motion (hence spatial distribution) of the charged carriers. The noise amplitude has a Gaussian distribution. In this case, using Carson's theorem, one can get $\overline{|i(\omega)|^2} = \dfrac{2q^2\tau^2kT}{ml^2(1+\omega^2\tau^2)}$ and

$S_i(f) \equiv = 2\overline{N_T}|I(2\pi f)|^2 = 2\overline{N_T}\dfrac{2q^2\tau^2kT}{ml^2(1+\omega^2\tau^2)} = \dfrac{4nVq^2\tau kT}{ml^2(1+\omega^2\tau^2)} = \dfrac{4kT}{R}\dfrac{1}{(1+\omega^2\tau^2)}$**,** where the rate of events, $\overline{N_T} = \frac{nV}{\tau}$, $V$ is the volume of the resistor, and $n$ the electron density. For practical cases, $\omega\tau \approx 0$, so the Spectral Density (good until ~$10^{13}$ Hz):

$$\boxed{S_v(f) = 4kTR}$$

Noise Voltage:

$$\boxed{\langle V_n{}^2 \rangle = \int_B S_v(f)\,df = 4kTRB}$$

where $\langle V_n^2 \rangle$ is the mean-squared thermal noise, $T$ is the absolute temperature in K, $B$ is the bandwidth, and $R$ is the real part of the impedance.

*Example*

$R = 1\ \text{k}\Omega$

$B = 100\ \text{KHz}$

$T = 300\ \text{K}$ or $v_n = 4\ nV/\sqrt{Hz}$ if without $B$.

$\Rightarrow \sqrt{\langle V_n^2 \rangle} = 1.3\ \mu\text{V}$

# <u>Flicker Noise</u> ( $1/f$ or pink noise)

In semiconductors this noise is mostly due to random trapping and detrapping of charges at the Si-SiO2 interface and associated changes in carrier mobility due to Coulombic scattering. Also, there's a limited distribution of the time constants associated with the trapping and detrapping. The noise (amplitude) is often non-Gaussian. Carbon resistor and MOSFET have Flicker noise. Metal-film resistor doesn't have it.

Theoretically, 1/f noise comes from a distribution of τ so approximately $S = \int S_i(\tau, f)d\tau \propto \int \frac{\tau}{(1+\omega^2\tau^2)} d\tau \propto \frac{1}{\omega} \propto \frac{1}{f}$. Empirically, equation like this is used all the time with proper measurement of the proportional constant.

$$\boxed{S_v(f) = K_f \frac{V^n}{f^m}}$$

$K_f$ is a constant associated with a device. $n$ is typically in the range of 0.5 and 2. $m$ is typically 1. The "$K_f$" can vary by several orders of magnitude so it's device-specific.

Proposed Hooge's formula

$$S_v(f) = \frac{\alpha_H}{N} \frac{V^2}{f}$$

# Flicker Noise in MOSFET

Two related mechanisms:

- Random trapping/detrapping of carrier at Si-SiO2 interface
- Change in bulk carrier mobility due to trapped charges and hence additional Coulombic scattering

Assuming uniform trap density at the interface, exponential electron wave decay in the oxide, n-MOSFET, and ignoring trap states far from $E_{Fn}$

$$S_{V_g}(f) = \frac{kTq^2}{f\gamma WLC_{ox}^2}(1+\alpha\mu N)^2 N_t(E_{Fn})$$

$$\gamma = \frac{4\pi}{h}\sqrt{2m^*\Phi_B}$$

$$\frac{1}{\mu} = \frac{1}{\mu_n} + \frac{1}{\mu_{ox}} = \frac{1}{\mu_n} + \alpha N_t(E_{Fn})$$

$W$ = Width of the channel

$L$ = Length of the channel

$\mu_n$ = Electron mobility without oxide charge scattering

$\mu_{\mathrm{ox}}$ = Electron mobility limited by oxide charge scattering

$S_{V_g}(f)$ = Noise Power Spectral Density

$\gamma$ = Attenuation function of electron wave in oxide $\approx 10^8\ \mathrm{cm}^{-1}$

$m^*$ = Effective mass of carrier in oxide

$\Phi_\mathrm{B}$ = Tunneling barrier height at the interface

$h$ = Plank's constant

$\alpha$ = Scattering coefficient $\approx 1*10^{-15}\ \mathrm{Vs}$

$N$ = Total number of channel carriers per unit area

$N_t$ = Number of interface traps

$E_{Fn}$ = Electron Quasi-fermi level

*For complete derivation see Hung, Ko, Hu, and Cheng, "A Unified Model for the Flicker Noise in Metal-Oxide-Semiconductor Field Effect Transistors", IEEE Trans. Elec. Dev., March 1990*

# Noise Bandwidth

$$\left\langle V_n^{\,2} \right\rangle_{in} = \int_0^{+\infty} S_v(f)\,df$$

For a system with transfer function $H$

$$\left\langle V_n^{\,2} \right\rangle_{out} = \int_0^{+\infty} S_v(f) \left| H(j2\pi f) \right|^2 df$$

$$\int_0^{+\infty} S_v(f) \left| H(j2\pi f) \right|^2 df \triangleq \int_0^{BN} S_v(f) \left| H(0) \right|^2 df$$

If white noise ($S_v(f)$ is constant),

$$\boxed{BN = \frac{1}{\left| H(0) \right|^2} \int_0^{+\infty} \left| H(j2\pi f) \right|^2 df}$$

# Noise Measures

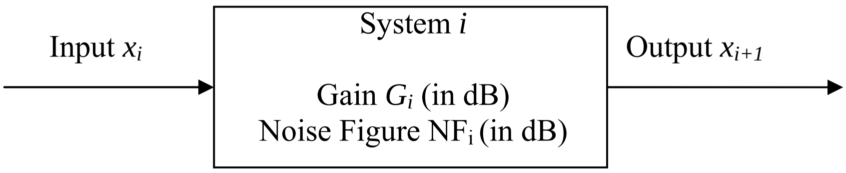
## Signal to Noise Ratio

$$\frac{S}{N} = \frac{\text{signal power}}{\text{noise power}}$$

Signal to noise ratio is often expressed in decibels (dB):

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

## Noise in Systems




$$F_i = \frac{\left(S/N\right)_{input}}{\left(S/N\right)_{output}} = \frac{\left(S/N\right)_{x_i}}{\left(S/N\right)_{x_{i+1}}}$$

$$x_{i+1} = G_i x_i$$

where $x$ is the signal power in dBm.

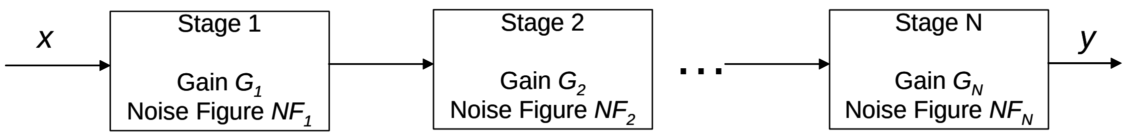
## Noise Factor ($F_i$)

$$NF_i = 10\log_{10}(F_i)$$

## Effective Noise Temperature ($T_i$)

$$T_i = 290\left(F_i - 1\right)$$

# Noise Factor of Cascaded Amplifiers




## Friis' Formula

$$F_{total} = F_1 + \sum_{i=2}^{N} \frac{F_i - 1}{\prod_{j=1}^{i-1} G_j}$$

*Example (N=3)*

$$F_{total} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

**Note: G must be its linear value, ie. not expressed in dB! F being the noise factor is also linear.**

# Noise In Common Devices

## Photoconductor (Neglecting shot and flicker noise)

Note that the photocurrent comes from excessive electrons and holes, and they are involved in a single time constant (τ) recombination mechanism as depicted in the equation.

$$\left\langle i_n^2 \right\rangle = 4kTGB + \frac{\tau}{t_r} \frac{4qBI_p}{1+\omega^2\tau^2}$$

$T$ = temperature in K
$G$ = conductance
$t_r$ = carrier transit time
$B$ = measurement bandwidth
$\tau$ = carrier lifetime
$\omega$ = light modulation frequency
$I_p$ = amplified photocurrent

## Forward-bias pn diode (Neglecting flicker noise)

Note that a diode should have shot noise and flicker noise. Modern diodes have been minimized in term of Flicker noise. If not, it should contain the Flick noise term. Also, if the series resistance is big, it should also contain the thermal noise ~4kTGB.

**In reverse bias**

$$\left\langle i_n^2 \right\rangle = 2qI_sB$$

**In forward bias**

$$\left\langle i_n^2 \right\rangle = 2qIB = 2qBI_s e^{\frac{qV}{kT}}$$

## Bipolar Transistor

$$\left\langle i_c^2 \right\rangle = 2qI_cB$$

$$\left\langle i_b^2 \right\rangle = 2qI_BB + K_1 \frac{I_B^a}{f} B$$

## FET Transistor

Note that $i_g$ is the gate leakage current. For the drain and/or source current, only $2/3g_m$ is counted for thermal noise because of geometric average.

$$\left\langle i_g^2 \right\rangle = 2qI_GB$$

$$\left\langle i_D^2 \right\rangle = 4KT(\frac{2}{3}g_m)B + 2qI_DB + K_1 \frac{I_D^a}{f} B$$