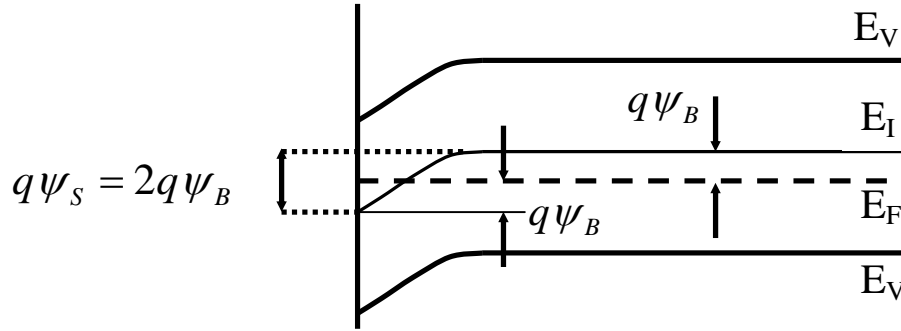


Threshold Voltage for an MOS Junction

At threshold, the concentration of electrons at the surface equal to the concentration of holes in the bulk: $n_{surface} = p_{p0}$



At threshold, $\psi_s = 2\psi_B$. It is assumed that all the charge in the semiconductor is due to the depletion region.

$$V_{TH} = V_{ox,T} + 2\psi_B = dE_{ox,T} + 2\psi_B$$

$$= d \frac{\epsilon_s}{\epsilon_{ox}} E_{s,T} + 2\psi_B$$

$$\text{Boundary: } \epsilon_s E_s = \epsilon_{ox} E_{ox}$$

$$= \frac{|Q_D|}{C_{ox}} + 2\psi_B$$

$$\text{By Gauss's Law } \epsilon_s E_s = |Q_D|$$

With $C_{ox} = \frac{\epsilon_{ox}}{d}$: Oxide Capacitance per unit Area

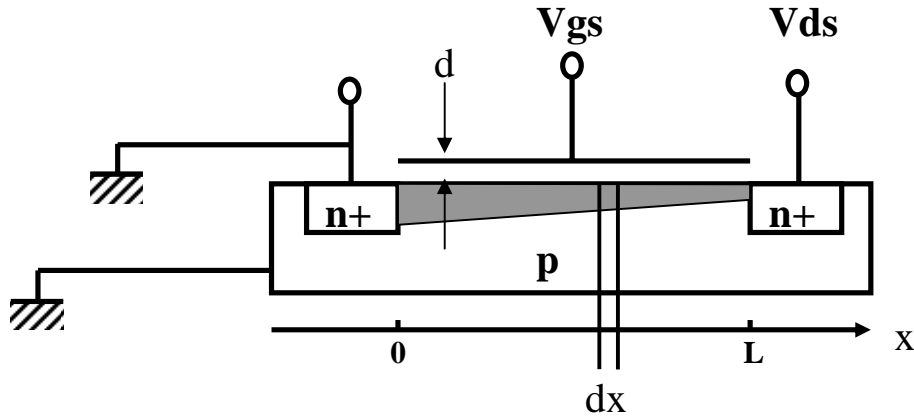
$$V_{TH} = \frac{|Q_D|}{C_{ox}} + 2\psi_B$$

For a real MOSFET, $V_{TH}' = V_{TH} - V_{FB}$

$$V_{TH} = \phi_{ms} - \frac{Q_{ss}}{C_{ox}} \pm 2|\psi| \pm \frac{|Q_D|}{C_{ox}} \quad (+ \text{ for p-type, } - \text{ for n-type})$$

where Q_{ss} is the charge trapped at the oxide/semiconductor interface.

Ohmic Mode (Triode)



$$Q_n(x) = C_{ox} (V_{gs} - V_{TH} - V(x)) \text{ With } C_{ox} = \frac{\epsilon_{ox}}{d}$$

$$V(0) = 0 \text{ and } V(L) = V_{ds}$$

Resistance of a length dx :

$$dR = \frac{\rho dx}{S} = \frac{dx}{\mu_n q n(x) S}$$

$$q n(x) S = \text{Charge per unit length in } x \text{ direction} = W Q_n(x)$$

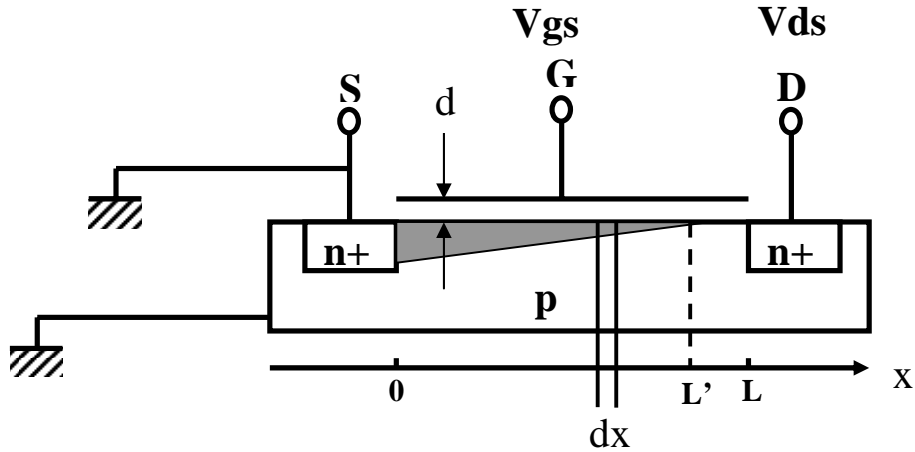
$$dR = \frac{dx}{\mu_n Q_n(x) W}$$

$$dV = I_D dR$$

$$\int_0^{V_{ds}} \mu_n W C_{ox} (V_{gs} - V_{TH} - V(x)) dV = \int_0^L I_D dx$$

$$I_D = \frac{W}{L} \mu_n C_{ox} \left[(V_{gs} - V_{TH}) V_{ds} - \frac{V_{ds}^2}{2} \right]$$

Saturation Mode (Pinch-Off)



$$qn(x) = C_{ox}(V_{gs} - V_{TH} - V(x)) \quad qn(L) = C_{ox}(V_{gs} - V_{TH} - V_{ds})$$

For $V_{ds} \geq V_{gs} - V_{TH} = V_{dssat}$, $n(L) = 0$: the channel is pinched-off near the drain

Further increase of V_{ds} doesn't increase the source-drain current

$$\int_0^{V_{dssat}} W\mu_n C_{ox}(V_{gs} - V_{TH} - V(x))dV = \int_0^{L'} I_d dx$$

$$I_d = \frac{W}{2L'} \mu_n C_{ox} (V_{gs} - V_{TH})^2 \cong \frac{W}{2L} \mu_n C_{ox} V_{dssat}^2 \equiv I_{dsat}$$

